Lecture 01 Introduction to Course

Oran Kittithreerapronchai¹

¹Department of Industrial Engineering, Chulalongkorn University Bangkok 10330 THAILAND

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OUTLINE

- CONTACT INFORMATION AND SYLLABUS
- Roles and Agreement
- MOTIVATION OF OPTIMIZATION
- Solving Parametric Univariate Optimization
- Supplement Materials for Intro to 2104529
 - Extended Concept and Roles of Optimization
 - Extended Concept and Roles of Data Mining
 - Solving Non-Parametric Univariate with Search

source: General references [NC20, TSK16, BG19, Pat14]

Syllabus: Before we start

Course Description

Industrial engineering problem solving using computational methods; data mining and visualizing; algorithms for inventory models, production planning, production scheduling, production analysis, and optimization

LEARNING OBJECTIVES



- Aware of a general computation method and optimization techniques in IE problems
 - Optimization (Unconstraint and Heuristic)
 - Algorithm & Data Mining, Visualization
- Applying suitable algorithm to IE problems
- Analyzing data for using large project

[e] [e]

REQUIREMENT

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- Aware of a general computation method and optimization techniques in IE problems [a]
 - Optimization (Unconstraint and Heuristic)
 - Algorithm & Data Mining, Visualization
- Applying suitable algorithm to IE problems
- Analyzing data for using large project

REQUIREMENT

- Good background in math modeling and optimization
- Decent programming skills, logic thinking, and perseverance



[e]

[e]

STILL HESITATE. WHY THIS ONE? WHAT FOR ME?

Why do need this course?

OKC's advisees

- M.Eng: computational and tool for thesis
- B.Eng: senior project, data analyst, advance comp skill

What are benefits of this course?

- Knowledge: unconstraint optimization, geom-calcu-algo, data mining, database
- R/RStudio: stand-alone/web app & service, wrapper, motivation for modern language
- Project: integration of thesis and senior project for team project

CONTACT INFORMATION

Name: Oran Kittithreerapronchai, PhD
Office: Room 603, Engineering Building 4

Office Hour: Tuesday, 16:00-17:00 or by appointment

Email: oran.k@chula.ac.th

Tel: 02-218-7781

BG & Expr: ACADEMIC: PhD dissertation, course, training,

 $\label{eq:professional} Professional: \ \mbox{analysis, stand-alone app, visualization}$

LMS: CourseVille (passwd: compMeth<<year>>)

WWW: http://ie.eng.chula.ac.th/~oran

https://sites.google.com/site/oranclasses/home

Grading Policy

GRADING

- Homework (40%)
- Midterm Exam (30%)
- Term Project: Data Mining (30%)

Grading & Scores

85 and above: final grade id definitely 'A'

between 50 and 85: A, B⁺, B, C⁺, ..., D

50 and below: final grade is possibly 'F'

CLASS RULES & AGREEMENTS

- No class attendance
- Don't interrupt others
- Focus on your term project
- Be responsible, especially meeting time and assignment
- Participate during class; this is Master level course
- Exam is designed to test student basic knowledge
- Term Project evaluates student performance

Code of honors

- Education with ethic standards and social responsibilities
- Trust as integral and essential part of learning process
- Self-discipline necessity
- Dishonesty hurts the entire community

adapted from: Georgia Institute of Technology –The Honor Code

Any violation to code of honors will severely punished, especially cheating and plagiarism

TEXTBOOK AND REFERENCES

Техтвоок

[NC20] Nwanganga, F. and Phapple, M. 2020. *Practical Machine Learning In R* Wiley

References

- [TOR16] Torgo, L. 2011 Data Mining with R: Learning with Case Studies. Chapman & Hall.
- [PAT14] Pathak, M. 2014 Beginning Data Science with R. Springer.
- [RS17] Ramasubramanian, K. and Singh, A. 2017 Machine Learning Using R: A comprehensive Guide to Machine Learning. Apress
- [TSK16] Tan P. at el 2016 Introduction to data mining. Pearson Education
 - [?] Chong, E. and Żak, S. 2013 An introduction to optimization John Wiley & Sons

ORGANIZATION

MATERIALS

- Main Tool: R/RStudio http://www.rstudio.com/ide/download/desktop
- Class website:
 - LMS: https://www.mycourseville.com
 - OKC: https://tinyurl.com/OKCWebpage/classes/CompMeth http://www.ie.eng.chula.ac.th/~oran/classes/CompMeth
- Other: Stack Overflow, GitHub, Datacamp

HOW TO TURN-IN YOUR ASSIGNMENT

- Medium: R markdown file and html
 - <ID>_hw<#homework>.Rmd (must be self-contain with RDS file)
- Where & When: through courseville and before midnight of due date

COMPUTATIONAL METHODS IN IE

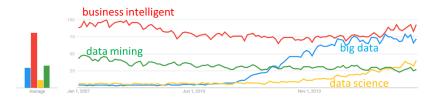
INDUSTRIAL AND SYSTEM ENGINEERING (ISYE)

- Core: applying systematic thinking to make "better" decisions
- Decisions: location, inventory, routing, slotting, planning
- Role of Data Mining in IE: understanding historical data
 - Target Micro Marketing: customizing promotion for each household (leaded)
 - Amazon Dynamic Pricing: setting price based on number of views (conceded)
 - Multi-Health System: a software company to decide whether a prisoner should get parol
 - Making a hit TV show: data mining approach for creating TV series (Ted Talks)
 - ullet Amazon make a competition of single episode of 8 candidate movie (comedy on four Rep. Senators) o Alpha house
 - \bullet Netflix analyze viewers, actors, producers, history (drama on single Rep. Senator) \to House of cards
- Role of Optimization in IE: achieving goals (cost, time, risk, regulation, and policies)

CONTENTS AND EXPECTATION

SOLVE IE PROBLEM WITH R

- Optimization: → familiar with R, explain algorithms
- Data Mining: → IE problem, Business analytic



Source. google.com/trend

- Experience, Time, and Effort are need for skills
- Homework is required to attend this class
- Contents is developing and evolving

If a company charges a price p USD for a product, then it can sell $3000e^{-p}$ units of the product. What is the revenue function?

• When does it decreasing/increasing in terms of p

- Is there any constraint?
- Find *p* that maximize the revenue

If a company charges a price p USD for a product, then it can sell $3000e^{-p}$ units of the product. What is the revenue function?

$$f(p) = 3000 \, p \, e^{-p}$$

• When does it decreasing/increasing in terms of p

- Is there any constraint?
- Find p that maximize the revenue

If a company charges a price p USD for a product, then it can sell $3000e^{-p}$ units of the product. What is the revenue function?

$$f(p) = 3000 p e^{-p}$$

• When does it decreasing/increasing in terms of p

$$\frac{\partial}{\partial p} f(p) = -3000 \, p \, e^{-p} + 3000 \, e^{-p} = 3000 \, e^{-p} (1-p)$$

increasing: 0

decreasing: 1

- Is there any constraint?
- Find p that maximize the revenue

If a company charges a price p USD for a product, then it can sell $3000e^{-p}$ units of the product. What is the revenue function?

$$f(p) = 3000 p e^{-p}$$

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$$\frac{\partial}{\partial p} f(p) = -3000 \, p \, e^{-p} + 3000 \, e^{-p} = 3000 \, e^{-p} (1-p)$$

increasing: 0

decreasing:
$$1$$

• Is there any constraint?

$$p \ge 0$$

• Find p that maximize the revenue

If a company charges a price p USD for a product, then it can sell $3000e^{-p}$ units of the product. What is the revenue function?

$$f(p) = 3000 p e^{-p}$$

When does it decreasing/increasing in terms of p

$$\frac{\partial}{\partial p}f(p) = -3000 \, p \, e^{-p} + 3000 \, e^{-p} = 3000 \, e^{-p}(1-p)$$

increasing: 0

decreasing:
$$1$$

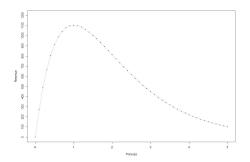
• Is there any constraint?

$$p \ge 0$$

Find p that maximize the revenue

$$\frac{\partial}{\partial p}f(p) = 0$$
 and solve for p
 $f(p = 1) = 1103.638$

Multiple ways to solve

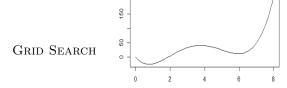


- Trial & Error: time consuming & solution quality \rightarrow always work
- ullet Graphic: dimensionality o provide understanding
- ullet Gradient: differentiable o exact solution
- Algorithm: combine all above all above



EXAMPLE

$$\min f_1(x) = x^4 - 14x^3 + 60x^2 - 70x, \ x \in \mathbb{R}$$



$$f_1(x) = x^4 - 14x^3 + 60x^2 - 70x$$

Calculus

Then
$$\frac{\partial}{\partial x} f_1(x) \equiv \frac{\partial^2}{\partial x^2} f_1(x) \equiv$$

##[5.957, 3.762, 0.781]

ALGORITHM

tempFn <- function(x) $\{x^4 - 14*x^3 +60*x^2-70*x\}$ optimize(tempFn,interval = c(0,2),maximum = F)

EXAMPLE

$$\min \ f_1(x) = x^4 - 14x^3 + 60x^2 - 70x, \ x \in \mathbb{R}$$

GRID SEARCH

$$f_1(x) = x^4 - 14x^3 + 60x^2 - 70x$$

Calculus

$$f_1(x) = x^4 - 14x^3 + 60x^2 - 70x$$
 Then $\frac{\partial}{\partial x} f_1(x) \equiv 4x^3 - 42x^2 + 120x - 70 = 0$ ##[5.957, 3.762, 0.781]

Algorithm

tempFn <- function(x) $\{x^4 - 14*x^3 +60*x^2-70*x\}$ optimize(tempFn,interval = c(0,2),maximum = F)

EXAMPLE

$$\min f_1(x) = x^4 - 14x^3 + 60x^2 - 70x, \quad x \in \mathbb{R}$$

GRID SEARCH

2 4 6 8
$$f_{\bullet}(y) = y^4 - 14y^3 + 60y^2 - 70y$$

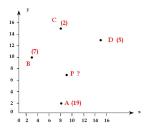
CALCULUS

$$\begin{array}{rcl} f_1(x) & = & x^4 - 14x^3 + 60x^2 - 70x \\ \\ \text{Then } \frac{\partial}{\partial x} f_1(x) & \equiv & 4x^3 - 42x^2 + 120x - 70 = 0 & \#\#[5.957, 3.762, 0.781] \\ \\ \frac{\partial^2}{\partial x^2} f_1(x) & \equiv & 12x^2 - 84x + 120 \end{array}$$

ALGORITHM

tempFn <- function(x) $\{x^4 - 14*x^3 +60*x^2-70*x\}$ optimize(tempFn,interval = c(0,2),maximum = F)

SINGLE FACILITY LOCATION PROBLEM

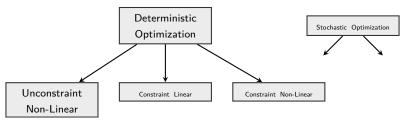


Source. MIT and James Orlin. 2003

Find the location of single warehouse that minimize the total distances from the following location. (assume all distances are 'Euclidean'.)

No.	customer	location (p_i)	$\#$ shipments (w_i)
1	Α	(8,2)	19
2	В	(3,10)	7
3	C	(8,15)	2
4	D	(14,13)	5

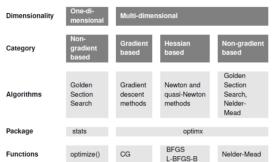
OPTIMIZATION TOPICS



- Univariate
- Multivariate

OPTIMIZATION TOPICS

OPTIMIZATION CLASS AND ALGORITHM

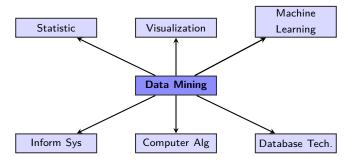


Optimization in R: https://cran.r-project.org/web/views/Optimization.html

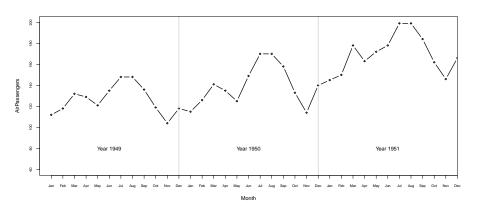
Supplement: math model cycle (see page 34), non-parametric (see page 41)

WHAT IS DATA MINING?

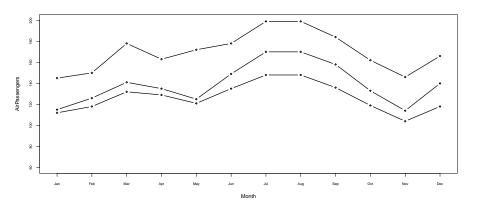
- What: process to discover interesting knowledge from data
- Tasks: prediction and descriptive (clustering)
- Why:
 - IT generates many data
 - computer power is cheap
 - business is high competitive



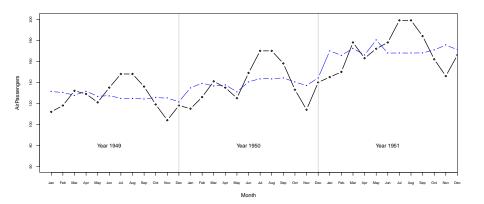
US AIR PASSENGERS 1949-1951



US AIR PASSENGERS 1949-1951



AIR PASSENGERS DE-SEASONALITY





FRAUD TRANSACTION IDENTIFICATION

- A company allows its to set price independently
- There are many products and saleperson
- Few percentage can be manually detect fraud

```
> data(sales)

> str(sales)

'data.frame': 401146 obs. of 5 variables:

SID : Factor w/ 6016 levels "v1"."v2"."v3"..: 1 2 3 4 3 5 6 7 8 9 ...

$ Frod : Factor w/ 4548 levels "p1"."p2"."p3".: 1 1 1 1 1 2 2 2 2 ...

$ Quant: int 182 3072 2093 112 6164 104 505 200 233 118 ...

$ Val : num 1665 8780 76990 1100 02060 ...

$ Insp: Factor w/ 3 levels "ok"."ufm"."fraud": 2 2 2 2 2 2 2 2 2 2 2 ...
```

Can you build a computer algorithm to identify the fraud?

FRAUD TRANSACTION IDENTIFICATION

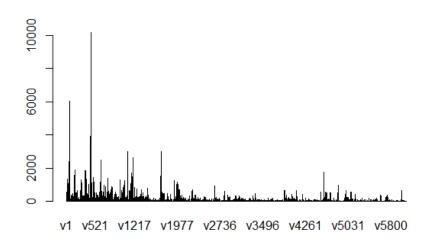
- A company allows its to set price independently
- There are many products and saleperson
- Few percentage can be manually detect fraud

```
> data(sales)
> str(sales)
'data.frame': d1146 obs. of 5 variables:
$ 10 : Factor w, 6016 levels "v1", "v2", "v3",..: 1 2 3 4 3 5 6 7 8 9 ...
$ Prod : Factor w, 4548 levels "p1", "p2", "p3",.: 1 1 1 1 1 2 2 2 2 2...
$ Quant: int 182 3072 2039 112 1614 04 350 200 233 118 ...
$ val : num 1665 $780 76990 1100 20260 ...
$ 1nsp : Factor w, 3 levels "ok", "unkn", "fraud": 2 2 2 2 2 2 2 2 2 2 2...
```

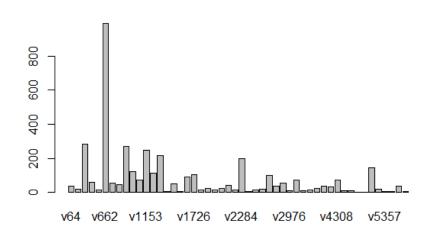
CAN YOU BUILD A COMPUTER ALGORITHM TO IDENTIFY THE FRAUD?

- What are differences between "ok" and "fraud"?
- Are unit price difference given ID, Prod, Insp?

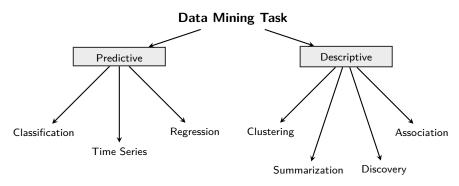
Overwhelming by Big Data



Overwhelming by Big Data



Data Mining Task



Data Mining in R: https://cran.r-project.org/web/views/MachineLearning.html

Supplement: knowledge discovery, data mining cycle (see page 38)

WHAT IS PARAMETRIC UNIVARIATE?

- Idea: known objective function single DV and no constraint
- Why: easiest problem
- Technique: Grid search, Stochastic search, First order condition, Newton method

EXAMPLE I

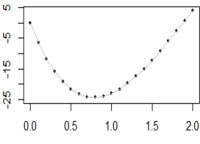
$$\min \ f_1(x) = x^4 - 14x^3 + 60x^2 - 70x, \ x \in \ \mathbb{R}$$

EXAMPLE II

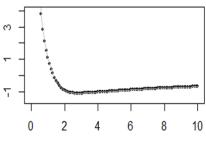
min
$$f_2(x) = 3xe^{-x^2} - 3\frac{1}{x}\ln(x), x \ge 0$$

GRID & STOCHASTIC SEARCH

• idea: systematically random/check domain and sort quality of solution



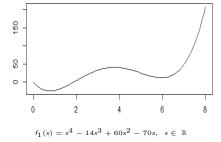
 $f_1(x) = x^4 - 14x^3 + 60x^2 - 70x, x \in \mathbb{R}$

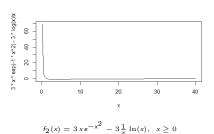


$$f_2(x) = 3xe^{-x^2} - 3\frac{1}{x}\ln(x), x \ge 0$$

GRID & STOCHASTIC SEARCH

• idea: systematically random/check domain and sort quality of solution





$$\begin{array}{rcl} f_1(x) & = & x^4 - 14x^3 + 60x^2 - 70x \\ \text{Then } \frac{\partial}{\partial x} f_1(x) & = & \\ & \text{and,} \\ \frac{\partial^2}{\partial x^2} f_1(x) & = & \end{array}$$

$$\begin{array}{rcl} f_2(x) & = & 3 \, x \, \mathrm{e}^{-x^2} - 3 \frac{1}{x} \ln(x) \\ \\ \text{Then} & \frac{\partial}{\partial x} f_2(x) & = \\ & \text{and,} \\ & \frac{\partial^2}{\partial x^2} f_2(x) & = \end{array}$$



$$\begin{array}{rcl} f_1(x) & = & x^4 - 14x^3 + 60x^2 - 70x \\ \text{Then} & \frac{\partial}{\partial x} f_1(x) & = & 4x^3 - 42x^2 + 120x - 70 \\ & \text{and,} \\ & \frac{\partial^2}{\partial x^2} f_1(x) & = & 12x^2 - 84x + 120 \end{array}$$

$$f_2(x) = 3xe^{-x^2} - 3\frac{1}{x}\ln(x)$$
Then $\frac{\partial}{\partial x}f_2(x) =$
and,
$$\frac{\partial^2}{\partial x^2}f_2(x) =$$

$$+ \frac{1}{x^3} - 2\frac{\ln(x)}{x^3}$$



$$\begin{array}{rcl} f_1(x) & = & x^4 - 14x^3 + 60x^2 - 70x \\ \text{Then} & \frac{\partial}{\partial x} f_1(x) & = & 4x^3 - 42x^2 + 120x - 70 \\ & \text{and,} \\ & \frac{\partial^2}{\partial x^2} f_1(x) & = & 12x^2 - 84x + 120 \end{array}$$

$$f_2(x) = 3xe^{-x^2} - 3\frac{1}{x}\ln(x)$$
Then $\frac{\partial}{\partial x}f_2(x) = 3e^{-x^2} - 6x^2e^{-x^2} - 3\frac{1}{x^2} + \frac{\ln(x)}{x^2}$
and,
$$\frac{\partial^2}{\partial x^2}f_2(x) = -6xe^{-x^2} - 12xe^{-x^2} + 12x^3e^{-x^2} + 6\frac{1}{x^3}$$

$$+ \frac{1}{x^3} - 2\frac{\ln(x)}{x^3}$$

TAYLOR SERIES FOR UNIVARIATE

$$q(x) = f(x_k) + f'(x_k)(x - x_k) + \frac{1}{2}f''(x_k)(x - x_k)^2$$

where, [Win22]

f(x) = Original objective function

q(x) = Taylor's approximated objective function

TAYLOR SERIES FOR UNIVARIATE

$$q(x) = f(x_k) + f'(x_k)(x - x_k) + \frac{1}{2}f''(x_k)(x - x_k)^2$$

where, [Win22]

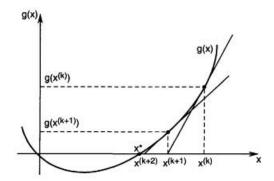
f(x) = Original objective function

q(x) = Taylor's approximated objective function

$$\frac{\partial}{\partial x}q(x) = f'(x_k) + (x - x_k)f''(x_k) = 0$$

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

WHAT IS NEWTON METHOD?



- What: iterative method by estimating a objective function with Taylor's series
- Advantage: efficient, basic for multiple decision variable
- Requirement: explicit function, twice continuity

Example: Newton Method I

Using Newton Method to find solution of this following function, if $x_0 = 0.0$.

$$f(x) = x^4 - 14x^3 + 60x^2 - 70x$$

- $f_1(x) = 4x^3 42x^2 + 120x 70$
- $f_1''(x) = 12x^2 84x + 120$

i	Xi	f(x)	$f'(x_i)$	$f''(x_i)$	x_{i+1}
0	0.0000	0.0000	-70	120.	$0 - \frac{-70}{120}$
1	0.5833	-23.079	-13.5	75.08	$0.5833 - \frac{-13.5}{75.08}$
2	0.7630	-24.360	-1.109	62.888	$0.7630 - \frac{-1.109}{62.888}$
3	0.7807	-24.369	-0.010	61.734	$0.7807 - \frac{-0.010}{61.734}$

When is it Time To Quit: Tolerance

What do we observe?

- **Solution:** virtually unchange, particularly $x_3 = 0.7807$ and $x_4 = 0.7808$
- **Objective Value:** virtually unchange, particularly $f(x_3) = -24.369$ and $f(x_4) = -24.370$
- **Slope:** virtually zero, particularly particularly $f'(x_3) = -1.109$ and $f'(x_4) = -0.010$
- Iterations: take too many iterations

Tolerance: ϵ

- Solution: $|x_{k+1} x_k| < \epsilon_x$
- Objective Value: $|f(x_{k+1}) f(x_k)| < \epsilon_{f(x)}$
- Slope: $f(x_{k+1}) < \epsilon_{f'(x)}$
- Iterations: t > N



EXAMPLE II: NEWTON METHOD

Using Newton Method to find solution of this following function using only the first derivative of $f_2(\cdot)$ start at x=1.0.

$$f_2(x) = 3xe^{-x^2} - 3\frac{1}{x}\ln(x)$$

•
$$f_2'(x) = 3e^{-x^2} - 6x^2e^{-x^2} - 3\frac{1}{x^2} + \frac{\ln(x)}{x^2}$$

•
$$f_2''(x) = -6xe^{-x^2} - 12xe^{-x^2} + 12x^3e^{-x^2} + 6\frac{1}{x^3} + \frac{1}{x^3} - 2\frac{\ln(x)}{x^3}$$

i	Xi	f(x)	$f'(x_i)$	$f''(x_i)$	x_{i+1}
0	1.0000	1.104	-4.104	6.793	$1.0000 - \frac{-4.104}{6.793}$
1	1.6041	517	-1.564	3.069	$1.6041 - \frac{-1.564}{3.069}$
2	2.1136	989	-0.442	1.341	$2.1136 - \frac{-0.442}{1.341}$
3	2.4432	-1.078	-0.137	0.584	$2.4432 - \frac{-0.137}{0.584}$

SUMMARY OF CLASS

2104529: R programming for some IE works

- IE Works: visualization, regression, decision tree, facility location
- who should take it? grad student, data cracking senior project
- why R, not python?: easy GUI and better help & support

Two Interrelated of Disciplines

- PART I: Unconstraint Optimization: theory and practical tools for solving optimization, e.g. FOC, Linear Algebra, Brent, Gradient method, Newton & Quasi-Newton methods
- PART II: Data Mining: computation tools using R for IE work, e.g., Decision Tree, Regression, Time Series

REFERENCE

[BG19] Brad Boehmke and Brandon M Greenwell. Hands-on machine learning with R. CRC press, 2019.

[NC20] Fred Nwanganga and Mike Chapple.

Practical machine learning in R.

John Wiley & Sons, 2020.

[Pat14] Manas A Pathak. Beginning data science with R. Springer, 2014.

[RS17] Karthik Ramasubramanian and Abhishek Singh. Machine learning using R. Number 1. Springer, 2017.

[Tor16] Luis Torgo.

Data mining with R: learning with case studies.

CRC press, 2016.

[TSK16] Pang-Ning Tan, Michael Steinbach, and Vipin Kumar. Introduction to data mining.
Pearson Education India, 2016.



GENERALIZATION OF MATH MODEL

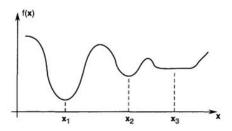
Let the problem have the general mathematical programming (MP) form

$$(P)$$
 minimize $f(\mathbf{x})$ subject to $\mathbf{x} \in \mathbb{F}.$

Mathematical programming

- ullet Yield a solution satisfying constraints o feasible solution (\mathbb{F})
- Solve for 'good' solution minimal value \rightarrow optimal solution ($f^*(\mathbf{x})$)

Local and Global Minimizer



Source. Chong & Zak. 2001 pp 72

LOCAL MINIMIZER

Suppose that $f \colon \mathbb{R}^n \to \mathbb{R}$ is a function defined on $\mathcal{F} \subset \mathbb{R}^n$. A point $\mathbf{x}^* \in \mathcal{F}$ is a *local minimizer* of $f(\cdot)$ if $\exists \ \epsilon > 0$ such that $f(\mathbf{x}) \ge f(\mathbf{x}^*)$ for all $\mathbf{x} \in \mathcal{F} \setminus \mathbf{x}^*$ and $\|\mathbf{x} - \mathbf{x}^*\| < \epsilon$

GLOBAL MINIMIZER

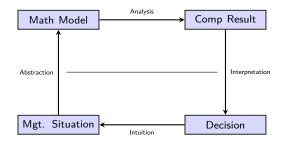
A point $x^* \in \mathcal{F}$ is a global minimizer of $f(\cdot)$ if $f(x) > f(x^*)$ for all $x \in \mathcal{F} \setminus x^*$

Components of Optimization Model

- Decision Variables: What are we interested in?
- Objective Function: How do we measure the "best" solution?
- Direction: minimize or maximize
- Constraints: What are the solutions?
- Parameters: Data needed to describe relationship

OPTIMIZATION CYCLE

Symbolic World



REAL WORLD

PATTERN DISCOVERY AND FEATHER SELECTION

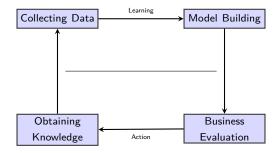
∃ thousands of patterns, yet none of them matter!

- Pattern Discovery Approach: better understanding, faster, cleaner
 - Suggested approach: Human-centered, query-based, focused mining
 - Data approach (Feature): filter method, wrapping method, embedded method
- interestingness measure
 - Objective: defined by statistics and structures of patterns, e.g., support, confidence
 - Subjective: defined by user belief in the data, e.g., unexpectedness, novelty.

Type of Data Mining

- **Data mined:** data warehouse, transactional, stream, spatial, time-series, text, multi-media, heterogeneous, WWW
- **Knowledge discovered (Task):** characterization, association, classification, clustering, trend/deviation, outlier
- Techniques utilized: machine learning, statistics, visualization
- Application adapted: retail, telecommunication, banking, fraud analysis

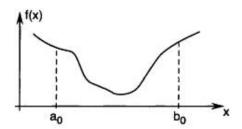
Data Mining Cycle



BACKGROUND

- What: smart trial-and-error of single variable
- Idea: reducing the search space/interval
- Issues: unimodal function, eliminate space/ interval
- Benefits: no objective function & computational expensive
- Advantage: solution quality, 'Smooth' function
- Examples: Grid Search, Golden section search, Fibonacci search

UNIMODAL FUNCTION



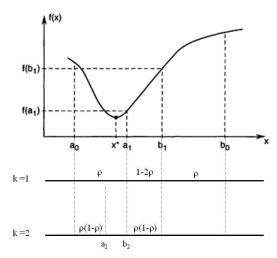
Source. Chong & Zak. 2001 pp 92

PROPERTIES

- Function has a single optima (minimal or maximal)
- Evaluating function require 3 points



GOLDEN SECTION SEARCH



Using Golden section search (3 iterations) to find solution between $\left[0,2\right]$ of this following function.

$$f(x) = x^4 - 14x^3 + 60x^2 - 70x$$

	LB		L.ce	enter	R.center		UB	
i	aį	$f(a_i)$	I_i	$f(I_i)$	r_i	$f(r_i)$	b_i	$f(b_i)$
1	0.000	0.000	.7639	-24.36	1.236	-18.96	2.000	4.000

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2	0.000	0.000	.4721	-21.10	.7639	-24.36	1.236	-18.96

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2	0.000	0.000	.4721	-21.10	.7639	-24.36	1.236	-18.96	
3	.4721	-21.10	.7639	-24.36	.9443	-23.59	1.236	-18.96	

Using Golden section search (3 iterations) to find solution between $\left[0,2\right]$ of this following function.

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i	a_i	$f(a_i)$	I_i	$f(I_i)$	r_i	$f(r_i)$	b_i	$f(b_i)$
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3	.4721	-21.10	.7639	-24.36	.9443	-23.59	1.236	-18.96
4	.4721	-21.10	.6525	-23.84	.7639	-24.36	.9443	-23.59