Lecture 06: Inventory Management

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Outline

1. Why do we have an inventory
2. Economic Order Quantity
3. Inventory Review System
4. Stochastic Demand with Newsvendor Model

Key Ref.: [JC10] [Bal07] [CM07] [Goe11]

What is Inventory?

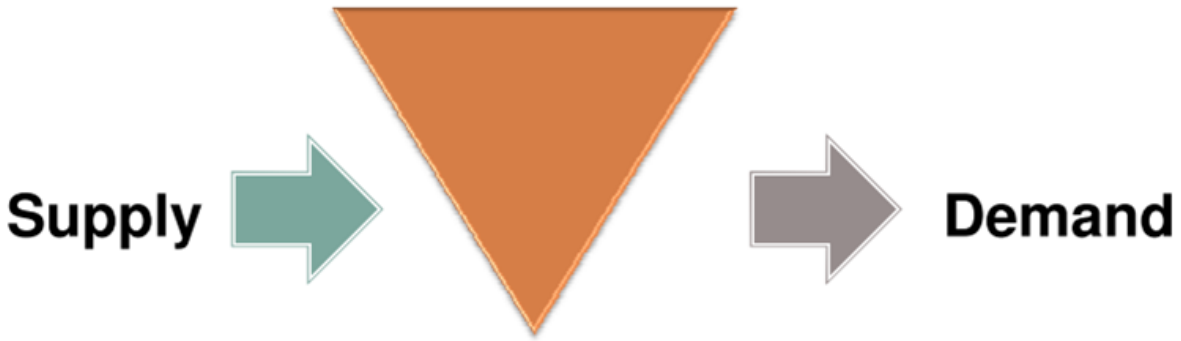
Inventory is a the stock of any item or resource used in an organization.

For example, raw materials, finished products, components, parts, supplies, and work-in-process (WIP)

Roles of Inventory

- To prevent variability in demands and delivery time
- To allow flexibility in production scheduling
- To take advantage of economic purchase-order size

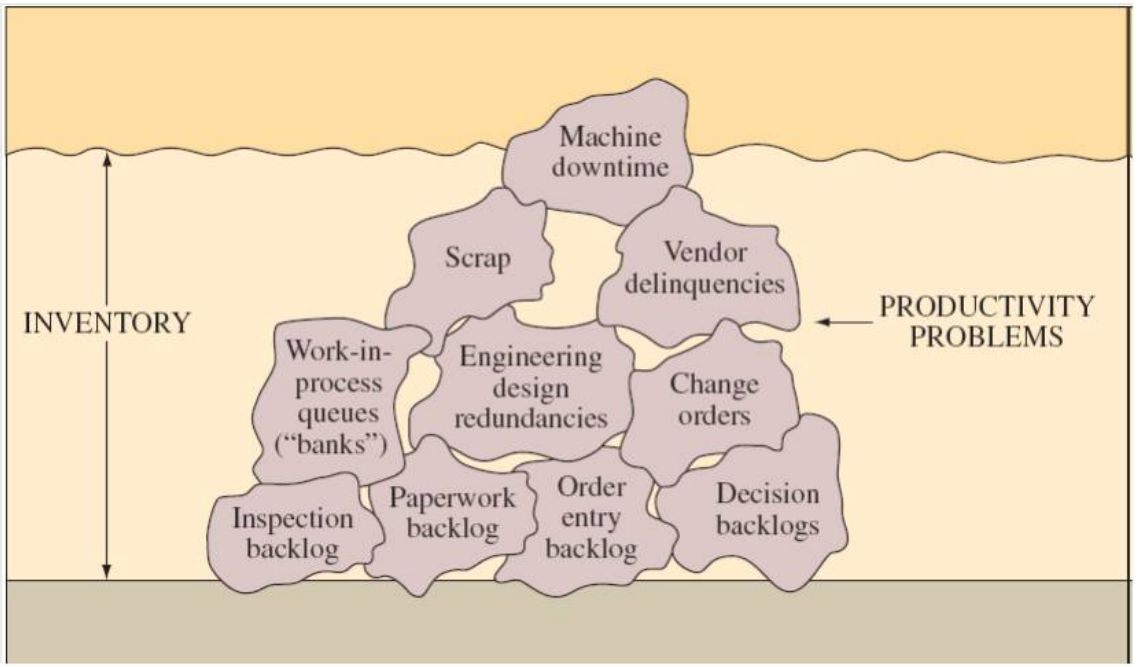
Demand ± Supply = Inventory



Type of Inventory

- **Pipeline inventory:** transportation, minimal lot-sizing, e.g., vessel/train
- **Buffer inventory:** imbalance of process, bottleneck
- **Seasonal inventory:** growth/slow down, speculation, e.g, BBQ grill, A/C
- **Cycle inventory:** regular consumption to exploit economy of scale
- **Safety inventory:** perspective risks

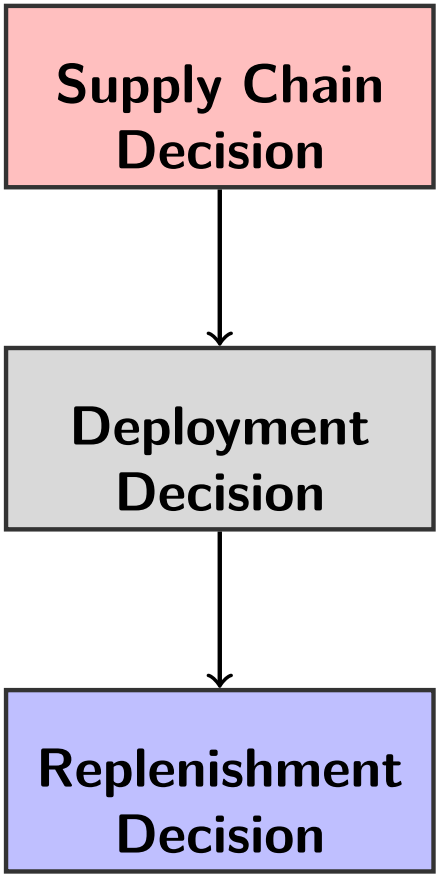
Why inventory is bad?



- **Capital:** opportunity (i.e., raw materials, labor, money), obsolete
- **Physical space:** need shelter and system to take care
- **Liability:** risk of theft/fire

Pg:Inventory Question

Hierarchy of inventory decision [**IE**]



- What are the potential alternatives?
- How should the product be designed?

- What SKUs should be carried as inventory?
- In what form should they be maintained?
- How much of each should be held and where?

- How often should inventory status be determined?
- When should a replenishment decision be made?
- What replenishment qty should be?

Main inventory control questions in IE

Replenishment Decisions

Given, what items need to stock with perfect information → review system

- When to order items from supplier or when to replenish?
- How much to order from supplier on each ordering

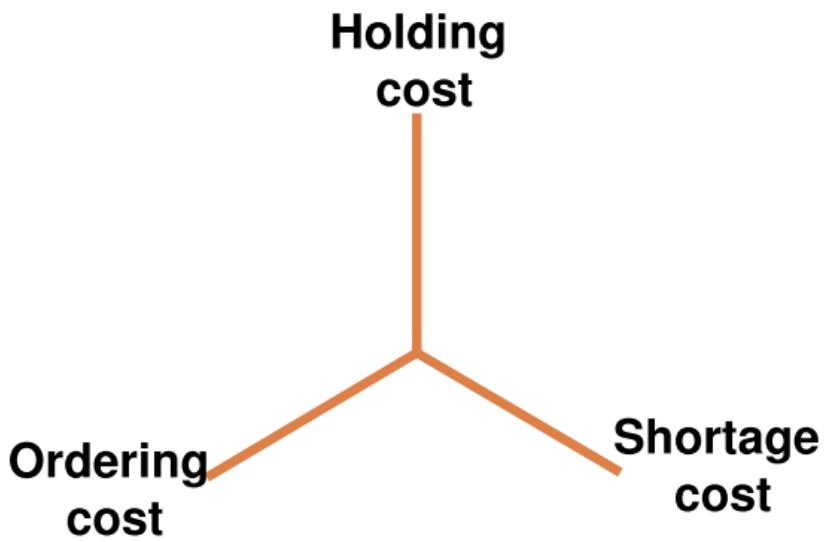
Goal:

Keep enough inventory to meet customer demand and also cost effective

Two Extreme Cases: Independent VS Dependent

- **Independent**: demands for various items are unrelated to others
- **Dependent**: demands for any one item is a proportional to others

Inventory costs



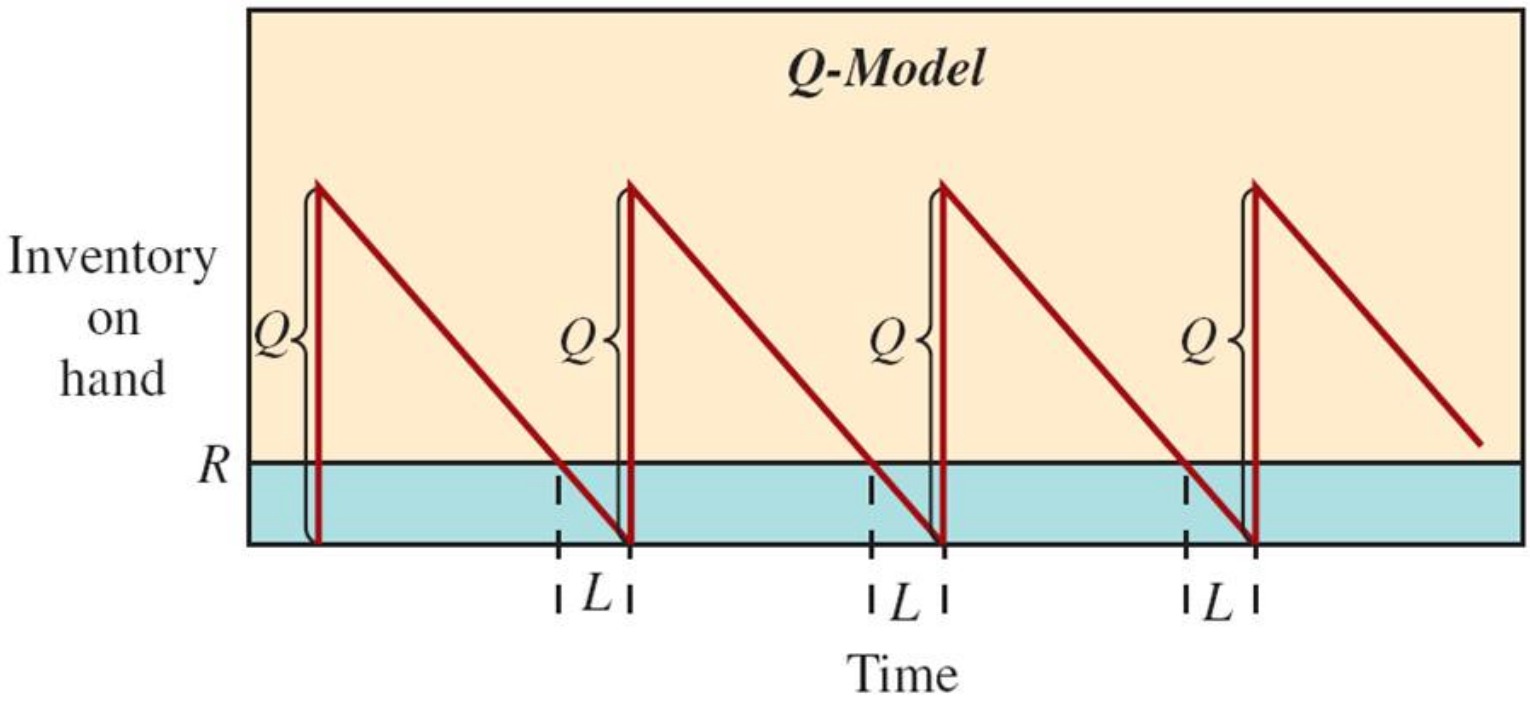
Holding costs costs for storage, capital, handling, spoilage, insurance

Ordering costs costs of placing an order (fixed & variable)

Shortage costs costs of running out, back-order, lost-sale

Economic Order Quantity: Simplest model

- single and independent product
- uniform (continuous) and constant demand
- instant replenishment and constant lead time



source: Chase and Jacob. 2011.

Derivation of EOQ

- **Parameters:**
  - Annual demand (D)
  - Order cost (S)
  - Unit handling cost (h)
  - Unit cost (c)
- **Decision variable:** order quantity (Q) ## why?
  - # Annual order $= \frac{D}{Q}$
  - Avg Inventory $= \frac{Q}{2}$

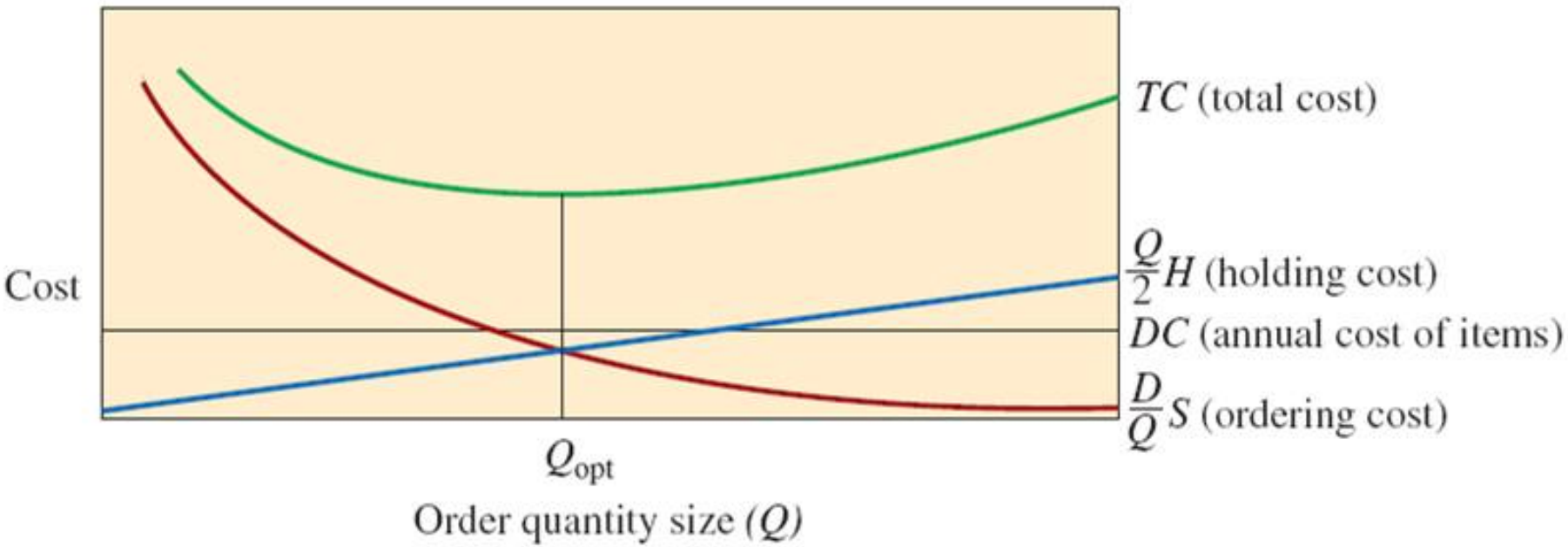
if demand = 240 unit/year, then

qty (Q)	1	10	30	40	120
# order	240	24	8	6	2

- **Total Costs:** material cost + ordering cost + holding cost

$$TC(Q) = \overbrace{cD}^{\text{material}} + \overbrace{S\frac{D}{Q}}^{\text{ordering}} + \overbrace{h\frac{Q}{2}}^{\text{holding}}$$

Holding cost VS Ordering cost



source: Chase and Jacob. 2011.

Derivation of EOQ

Economic Order Quantity

$$\text{Quantity } (Q^*) = \sqrt{\frac{2D\,S}{h}}$$

$$\text{Total Cost } (TC(Q^*)) = cD + \sqrt{2D\,S\,h}$$

where, Q = Quantity

$TC(Q)$ = Total costs

D = Demand rate

c = Unit purchasing Cost

S = Ordering cost

h = Holding cost

Example of EOQ

At Allied Electric (HK), The expected annual demand of Sumsung TV is 12,000 units. Administrative, transportation, and receiving form the fixed cost of $4000 each time an order is places. Each TV cost $500 to buy and the annual holding cost is 20 % of value. Find EOQ

source: Chopra and Meindl. 2010 example 10.1

Parameters

- **Demands (D):**
- **Unit cost (c):**
- **Ordering Cost (S):**
- **holding Cost (h):**

EOQ Insight to inventory control

- Trade-off between holding cost and ordering cost

Key insight

Total holding cost = Total ordering cost

- Q^* unaffected by unit cost of product
- Q^* can be reduced by
  - decreasing ordering cost
  - increasing handling cost
- Using EOQ creates bullwhip effect

Limitation of EOQ: rank from possible to impossible

- instant replenishment
- nearly constant demand
- zero and constant lead time

Academic Extension of EOQ Model [**IE**]

2104611: Inventory Analysis

Production Replenishment is not instantaneous

Multiple Products Combine dependent products to share ordering cost

Multi-echelon Upstream EOQ is multiple of downstream EOQ

Safety stock Additional quantity to protect surging in demands

Next Example

Quantity Discount Buy more to reduce total costs

Storage Constraint Warehouse has limited storage space

Example: Ordering carton

A bottler company must order 30,000 corrugated boxes from a cardboard suppler, annually. Currently, the suppler sells cardboard at 10 THB per box and charges a delivery cost at 400 THB per transaction. If the company estimate its internal handling costs at 15% of its values. Answer the following questions

A. What is the EOQ and total logistics costs?

B. If the company has only 2000 boxes storage space within it compound the factory, how is the solution changed?

C. If the company stores additional boxes at an outside warehouse at cost 1 THB per box per year, how is the solution changed?

D. If the suppler waives the delivery charge when the company orders more than 30,000 THB per order, how is the solution changed?

Solution: Ordering carton

Parameters

- **Demands (D):**
- **Ordering Cost (S):**
- **holding Cost (h):**

Solution

A) **EOQ (Q^*):**

B) **Limited Space:**

C) **Rent:**

D) **Free shipping:**

Can EOQ apply to real world?

- **Demand:** stochastic → known distribution
- **Lead time:** constant(qty)/random
- **How:** continuous review of current inventory
- **When:** enough for expected sale till get a new order
- **Safety Stock:** inventory for randomness till waiting a new order

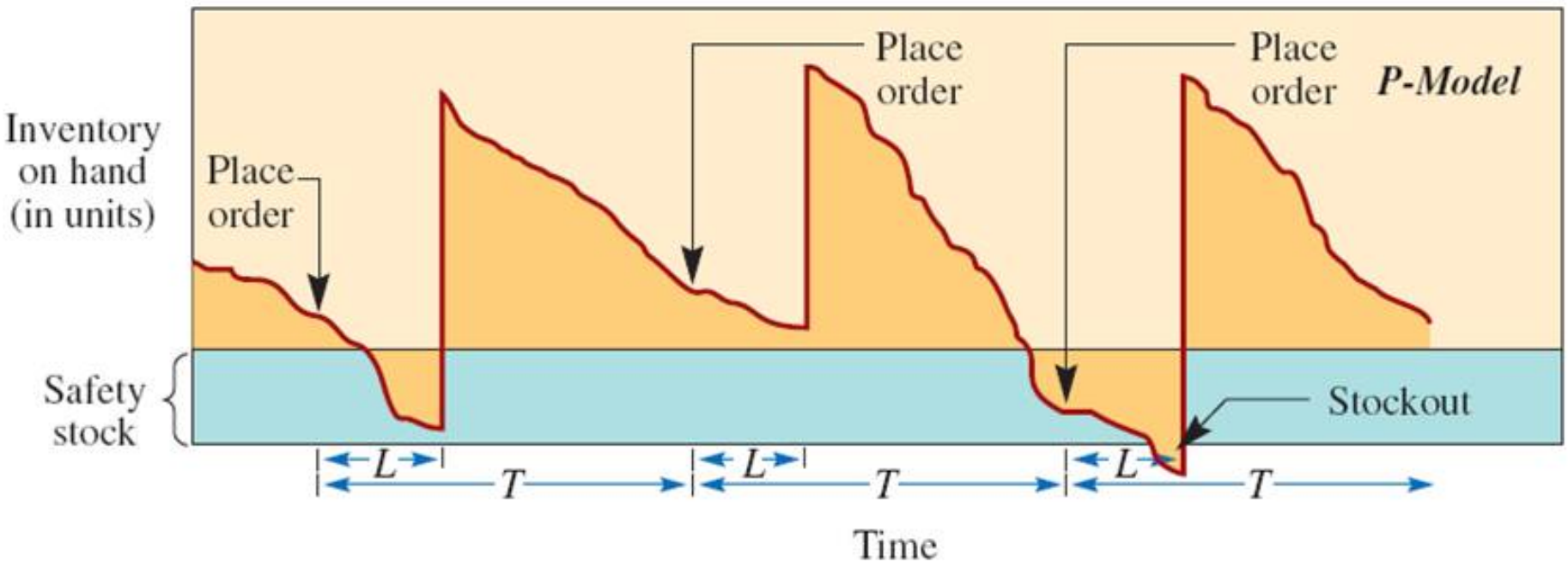
Example of Policy [**IE**]

(Q, q) policy is used for order

- q: reorder point, expected demand during lead time + safety stock
- Q: order quantity derived from EQO model (using expected sale) + safety stock

- $\exists$ chance of out-of-stock → service level

Real world: Random demand and lead time



source: Chase and Jacob. 2011.

Safety stock

- **Idea:** demand is unpredictable; if $\nexists$ buffer inventory, lost sale
- **What:** buffer inventory for unexpected demands during ordering
- **Factors** variability of demands and lead time
- **Formulation:** safety stock = service level $\times$ variation

$$\text{avg. demand}_{LT} = \bar{D}\,\bar{L} \quad ; \quad \text{var. demand}_{LT} = \bar{L}\,\sigma_D + \bar{D}^2\,\sigma_L$$

- **De-mystify:** safety stock $\neq$ demands during lead time
- **Reduce safety stock:** priority to reduce safety stock
  - reduce variability of lead time
  - reduce expected lead time
  - reduce variability of demand

How often inventory should be determined?

Periodic review system

Pg: Periodic Review

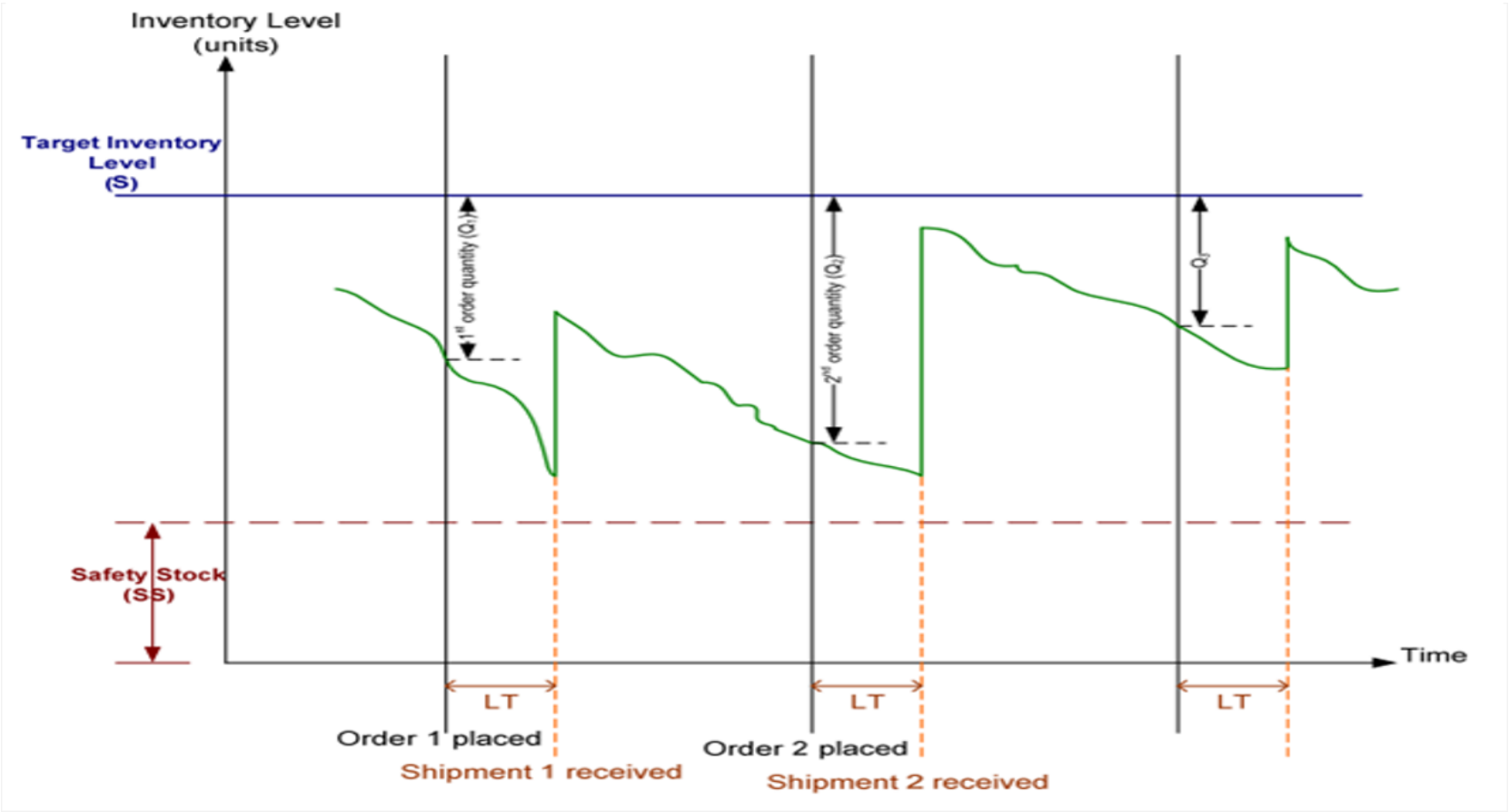
- **Idea:** checking inventory at regular period
- **Example:** end-of-fiscal year
- **Pro:** no tool, less effort to review
- **Evidence:** order duration

Continuous review system

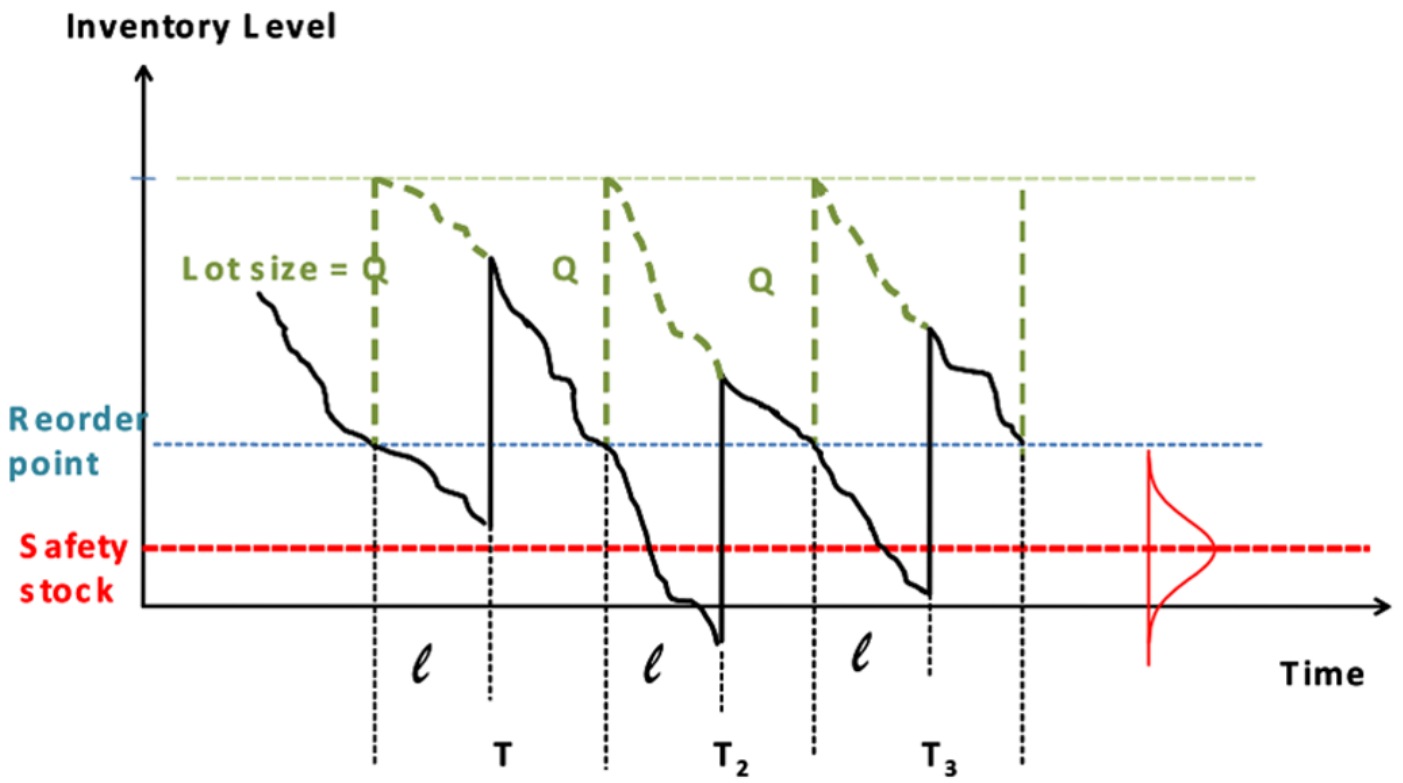
Pg: Continuous Review

- **Idea:** continuously tracking inventory
- **Example:** two-bin system
- **Pro:** less avg inventory
- **Evidence:** order date, trigger qty level
- **Note:** Tool=IT to improve speed and accuracy

Periodic review [**IE**]



Continuous review [**IE**]



Do we have good inventory management?

Symptoms of bad inventory control

- insufficient inventory
- non-moving/ dead stock/ obsolete
- inaccurate stock

KPI

- **Fill rate:** $\frac{\text{product shipped}}{\text{product order}}$
- **Inventory turnover:** $\frac{\text{annual sale}}{\text{inventory}}$
- **Inventory accuracy:** $1 - \frac{|\text{actual} - \text{account}|}{\text{actual}}$

Inventory turnover

	2011	2010	2009	2008	2007
Kohl's					
Revenue	18391	17178	16389	16474	15544
Cost of Goods Sold	11359	10679	10332	10459	9890
Inventory	3036	2923	2799	2856	2588
Net Income	1114	991	885	1084	1109
Walmart					
Revenue	418952	405046	401244	374526	344992
Cost of Goods Sold	307646	297500	299419	280198	258693
Inventory	36318	33160	34511	35180	33685
Net Income	16389	14335	13188	12884	12036

$$\text{Inventory turnover} = \frac{11359}{3036} = 3.74$$

Perishable Products

- **Product:** good short period and single order
- **Example:** newspaper, magazines, flowers, produces, seasonal clothing

Example

A newspaper costs (c) \$1 and can be sold for (p) \$2 with savage value (v) zero. What is the profit ($\pi(q)$). if we order (q) 20 today and demand (d) is:

Demand (d)	Sold	Leftover	Lost sale	Profit (π)
20	20	0	0	$2(20) - 1(20) + 0(0) = \$20$
15	15	5	0	$2(15) - 1(20) + 0(5) = \$10$
30	20	0	10	$2(20) - 1(20) + 0(0) = \$20$

$$\pi(q) = \overbrace{p \times \min(d, q)}^{\text{revenue}} - \overbrace{c \times q}^{\text{purc.cost}} + \overbrace{v \times max(0, q - d)}^{\text{savage value}}$$

Optimal Newsvendor

Newsvendor

$$
\begin{aligned}
\text{Expected total Profit} &= \text{Expected net revenue} - \text{Total net investment} \\
\pi(q) &= (p-v)\cdot S(q) + v\cdot \mu(q) - c\cdot q \\
\text{Quantity } (q^*) &= F^{-1}\left(\frac{p-c}{p-v}\right) = F^{-1}\left(\frac{p-c}{p-c+c-v}\right) \\
&= F^{-1}\left(\frac{C_u}{C_u+C_o}\right)
\end{aligned}
$$

where,
- $\pi(q)$ = Total profit
- $F(\cdot)$ = Cumulative probability function
- $S(q)$ = Expected units sold, $S(q) = q - \int_0^q F(y)dy$
- $\mu(q)$ = Expected units unsold, $\mu(q) = \int_0^q F(y)dy$
- C_u = Cost of under stock, $C_u = p - c$
- C_o = Cost of overstock, $C_o = c - v$

Example of Newsvendor

Sportmart needs to determine quantities of skis for the winter. Based on historical demand, the demand of ski is normally distributed with mean (μ) 350 and sd (σ) 100. Each pair of ski costs \$100 and retail for \$250. The unsold skis are disposed for \$85 assuming it costs \$5 to hold inventory for the entire winter.

source: Chopra and Meindl. 2010 example 12.1

Steps

0 **Model:** Newsvendor model $\mathcal{N}(\mu = 350, \sigma = 100)$

1 **Parameter:** $p = \$250,\ c = \$100,\ v = 85 - 5 = \$80$

$$C_u = 250 - 100 = \$150 \text{ and } C_o = 100 - 80 = \$20$$

2 **Pr. Sell:** $F(q^*) = \frac{150}{150+20} = 0.88$

3 **Quantity:** $q^* = 350 + 100z^{-1}(0.88) = 468$

Insight to Inventory Control

- Trade-off between opportunity cost and overstocking cost
- q^* is equilibrium overage and underage costs are balanced
- Assuming $p > c > v$
- $\frac{C_u}{C_u+C_o}$ is between $[0, 1]$ similar to probability
- Large C_u comparing to $C_o \rightarrow$ huge purchase
- Small C_u comparing to $C_o \rightarrow$ little purchase

Reference

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European Business Review, 19(4):332–348, 2007.

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Operations and supply management: The core.
McGraw-Hill Irwin New York, NY, 2010.

How to play Pharmaceutical Inventory Game

																	Supplier 1				Supplier 2			Supplier 3							
																	A	B	C		C	D									
															หน่วยการสั่งเป็นจำนวนเท่าของ				25	25	10		20	50		Flexible on Order					
															ขนาดสั่งขั้นต่ำที่จะยอมส่งสินค้าให้		MOQ		75	50	50		40	100		Flexible on Order					
		ความต้องการรวมเมื่อสิ้นวัน (ชิ้น)					Incoming Recieve					ปริมาณพัสดุในคลัง เมื่อสิ้นวัน(ชิ้น)				Order Supplier 1				Order Supplier 2			Special Order Supplier 3							
Period No	DAY	A	B	C	D		A	B	C	D		A	B	C	D	A	B	C	SUM	C	D	SUM	A	B	C	D	SUM				
0												50	50	80	80				0			0					0				
1	Mon						0	0	0	0		50	50	80	80	200	150	100	450	0	200	200					0				
2	Tue						0	0	0	0		50	50	80	80				0			0					0				
3	Wed						0	0	0	200		50	50	80	280				0			0					0				
4	Thu						0	0	0	0		50	50	80	280				0			0					0				
5	Fri						200	150	100	0		250	200	180	280				0			0					0				

https://tinyurl.com/ChulaInvGame

Historical Data

What are products in the game?

Important Insights: Cost VS Service

- **Focus on what matter:** many not important; only 20% matter
- **Opportunity Cost:** inventory cost = opportunity, not real money
- **Information = Power:** demand forecasting, parameter accuracy
- **Product Matter:** know products, nature/relationship, regulation (GDP)
- **Move to next phrase:** not just money, but customer, internal growth

Quick-win in inventory management:

- **Purchasing Power:** use IT or contracts, e.g, VMI, Collaborative Purchasing (CP), paid-per-used
- **Sharing & Borrowing:** build alliance, pool risk with visibility
- **Right Dispensary Unit:** blister packs, box, bottle